HWRS 505: Vadose Zone Hydrology

Lecture 5

9/5/2023

Today:

1. Air-water system in capillary tubes
2. Model of a porous medium: Bundle of Capillary Tubes

Art of porous media flow

Optional, but strongly encouraged, Mini-project

Take a photo or a video (< 2 min) in your daily life that you think best illustrates some cool phenomena of porous media flow.

I will create a dropbox on D2L for you to upload the photo or video (due on November 27th).

Depending on the quality of your picture or video, you can receive up to 5 bonus points in your final grade (out of 100 points).

Air-water system in capillary tubes

Review of Lecture 4

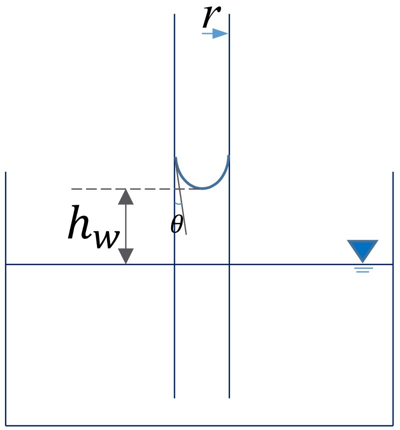
- ❖ Solute transport
  - ❖ Flux law for solute transport in porous media.
  - ❖ Derivation of advection-dispersion equation.
- ❖ Air-water system in a capillary tube
  - Interfacial tension
  - Wettability, contact angle
  - <u>Capillary pressure</u>, <u>Young-Laplace Equation</u>

Pressure jump across a fluid-fluid interface

Pressure jump across a fluid-fluid interface is determined by interfacial tension + geometry of the interface (radii of the curvature)

Air-water system in capillary tubes

Capillary rise in a Capillary tube



At the water table:

$$\left.\begin{aligned} P_a &= 0 \\ P_w &= 0 \\ z &= 0 \end{aligned}\right\} \Rightarrow H = 0$$

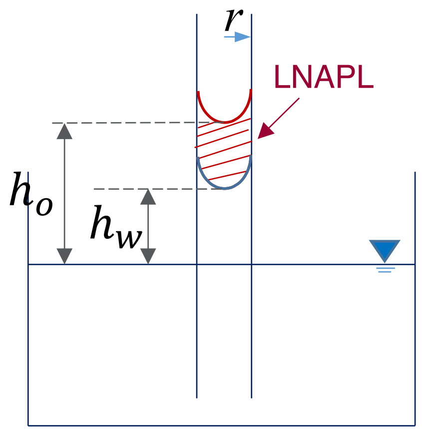
At the air-water interface in the tube:

$$\left.\begin{aligned} P_a &= 0 \\ z &= h_w \\ H &= 0 \end{aligned}\right\} \Rightarrow \phi = -h_w \Rightarrow P_w = -\rho_w g h_w$$

$$\left.\begin{aligned} &p^{cap} = P_a - P_w = 0 - (-\rho_w g h_w) = \rho_w g h_w \\ &\text{Young\_Laplace Eqn: } p^{cap} = \frac{2\sigma_{aw}\cos\theta}{r} \end{aligned}\right\} \Rightarrow h_w = \frac{2\sigma_{aw}\cos\theta}{\rho_w g r}$$

Air-water system in capillary tubes

Capillary rise in a Capillary tube in the presence of an LNAPL (Assuming zero contacts)



note: w/o oil ($\theta = 0$)

$$h_w = \frac{2\sigma_{ow}}{\rho_w g r}$$

At the oil-water interface:

$$P_w = 0 - \rho_w g h_w = -\rho_w g h_w \quad \text{---} \quad (1)$$

$$P_o = P_{ow}^{cap} + P_w = \frac{2\sigma_{ow}}{r} - \rho_w g h_w \quad \text{---} \quad (2)$$

continuing through the oil to the oil-air interface, the oil pressure

$$P_o = \frac{2\sigma_{ow}}{r} - \rho_w g h_w - \rho_o g (h_o - h_w) \quad \text{---} \quad (3)$$

At the oil-air interface:

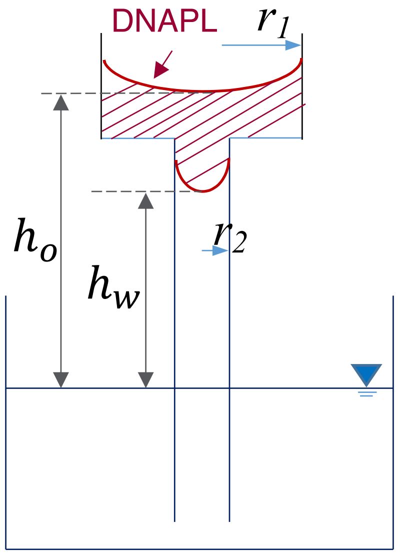
$$P_o = 0 - P_{oa}^{cap} = -\frac{2\sigma_{oa}}{r} \quad \text{---} \quad (4)$$

P_o in (3) & (4) are the same $\Rightarrow$

$$h_w = \left(\frac{2\sigma_{ow}}{r} + \frac{2\sigma_{oa}}{r} - \rho_o g h_o\right) / (\rho_w g - \rho_o g)$$

Air-water system in capillary tubes

Invasion of a nonwetting fluid into a pore (Assuming zero contacts)



At the oil-water interface:

$$P_o = P_a - P_{oa}^{cap} = -\frac{2\sigma_{oa}}{r_1}$$

continuing down to the oil-water interface, the pressure of oil is:

$$P_o = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g$$

At the oil-water interface:

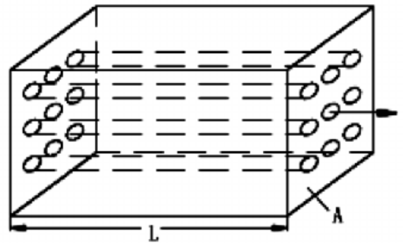
$$\begin{cases} P_{ow}^{cap} = P_o - P_w = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g + \rho_w g h_w \\ P_{ow}^{cap} = \frac{2\sigma_{ow}}{r_2} \end{cases}$$

$$\Rightarrow \quad h_o - h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_w g h_w\right] / (\rho_o g)$$

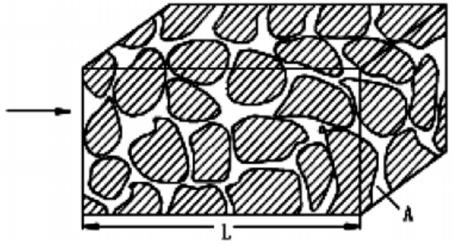
$$\Rightarrow \quad h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_o g h_w\right] / \left[(\rho_w - \rho_o) g\right]$$

Air-water system in capillary tubes

Model of a porous medium as a Bundle of Capillary Tubes

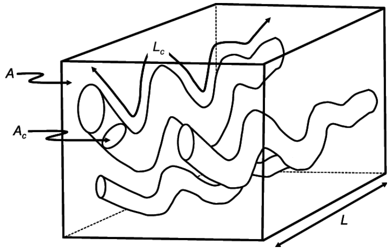


Bundle of capillary tubes



porous medium

Bear (1972)

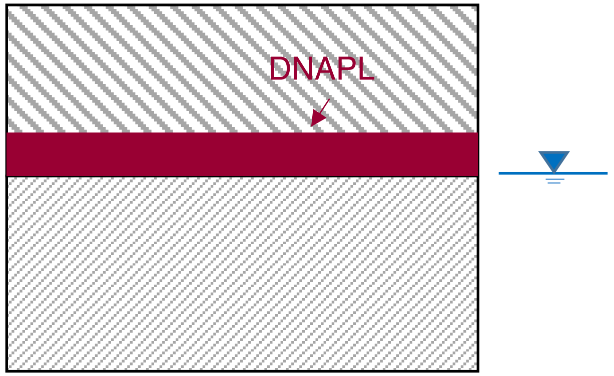


Bundle of tortuous capillary tubes

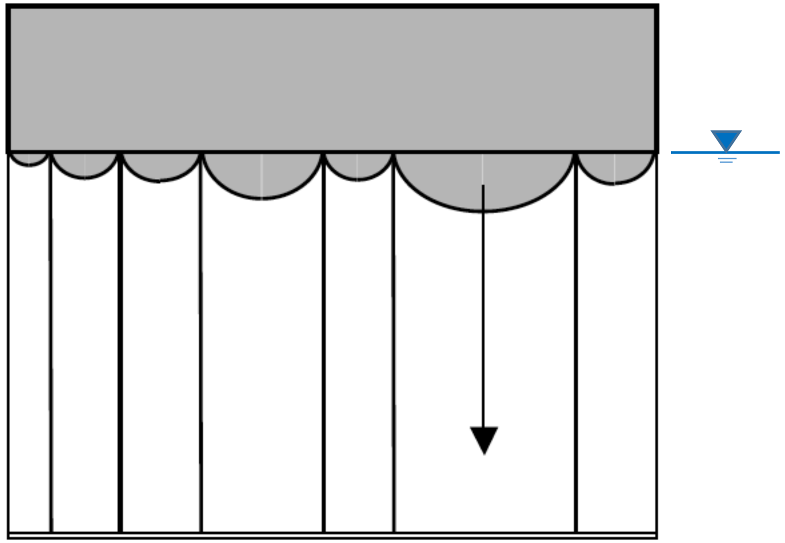
- Very simplified model, but its application has tremendously improved our understanding of fluid flow and transport phenomena in porous media. Some examples:
  - Permeability (already discussed)
  - Dispersion (already discussed)
  - Fluid invasion
  - Capillary transition zone
  - Soil water characteristic curve
  - Relative permeability

Air-water system in capillary tubes

Invasion of a nonwetting fluid into an aquifer



Representing the aquifer as a bundle of capillary tubes



1. Which is easier for DNAPL to invade?

   Coarse sand or fine-grained medium?

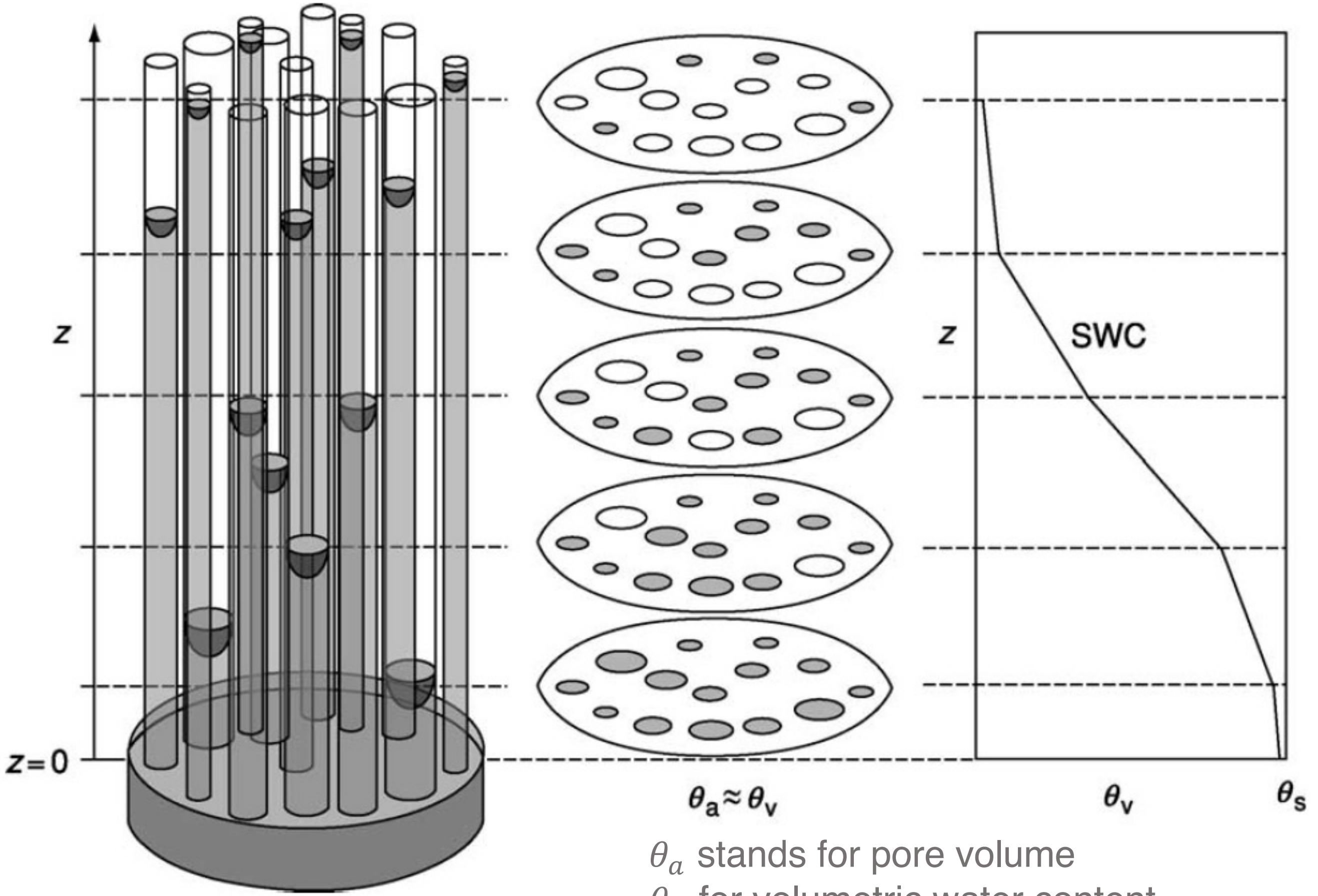
2. For some reason, if DNAPL modifies the wettability of the porous medium grain surfaces, e.g., the contact angle of water increases from 0^0 to something between 0^0 and 90^0.

   What may happen to the DNAPL?

Air-water system in capillary tubes

Water retention (or capillary transition zone) in the vadose zone

Representing the soil as a
Bundle of Capillary Tubes



Tuller & Or (2004)

θ_a stands for pore volume
θ_V for volumetric water content
θ_s for volumetric water content at saturation