## HWRS 561b Homework #3

- Assigned: Sunday, 22 February 2026
- Due: Sunday, 8 March 2026 (upload answers in PDF or Jupyter Notebook to D2L)
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- Teaching Assistant: Aldo Andres Tapia Araya
- Semester: Spring 2026

1. (10 points) Propose two questions that you think can be used as midterm oral exam questions. The questions should focus on some of the most critical concepts that we have covered so far in this course. Keep in mind that these are oral exam questions, so they should not involve detailed calculations; rather, they should focus on testing whether one can draw or develop conceptual pictures and formulate physical arguments. Make sure to describe your questions concisely and accurately. Ask one of your classmates to answer your questions. Then, summarize her/his answers below. Please also write your own comments on the answers. You should also put down her/his name.

2. (20 points) Consider a confined aquifer into which an LNAPL has been spilled. The LNAPL accumulates below the upper aquitard. The aquifer and aquitard materials are water-wet with contact angle $\theta = 0°$. A representative pore radius for the aquitard is 0.05 mm. A representative pore diameter for the confined aquifer is 1 mm. The hydraulic head of water in the aquitard (above the LNAPL) is identical to the hydraulic head of water in the aquifer (below the LNAPL), and water is hydrostatic everywhere. The densities of water and LNAPL are 1.0 and 0.975 g cm$^{-3}$, respectively. The interfacial tension between LNAPL and water is 0.02 N m$^{-1}$.

   - (a) (10 points) Draw a capillary-tube representation of this system. (Hint: approximate both aquifer and aquitard as bundles of tubes; for simplification, one tube for the aquifer and one tube for the aquitard is sufficient.)
   - (b) (10 points) Write a general expression (in terms of $r_{\text{aquitard}}$, $r_{\text{aquifer}}$, $\rho_{\text{water}}$, $\rho_{\text{LNAPL}}$, $\sigma_{\text{water-LNAPL}}$, and $h_{\text{LNAPL}}$) for the maximum LNAPL thickness that can exist below the aquitard before LNAPL first enters the aquitard. Then use the values above to compute that thickness.

3. (20 points) Use the bundle of cylindrical capillary tubes to complete the following two problems.

   - (a) (10 points) The relative permeability function is almost always nonlinear. Examples are the Brooks-Corey and van Genuchten models. Explain why the relative permeability function is nonlinear. Additionally, explain how the relative permeability functions for wetting and nonwetting fluid phases are different and why.
   - (b) (10 points) Explain the algorithmic steps to derive a relative permeability curve if an SWC is measured for a soil. You are not allowed to use any existing functional forms for the relative permeability.

4. (20 points) Unsaturated flow and relative permeability.

   In 1D vertical flow (positive $z$ upward) for a homogeneous medium, use extended Darcy's law for the wetting phase:

$$q_w = -K_{sat} k_{rw}(S_e) \left( \frac{dh}{dz} + 1 \right).$$

Use the Brooks-Corey-Mualem type relation

$$k_{rw} = S_e^{(2+3\lambda)/\lambda}.$$

Parameters: $\theta_r = 0.08$, $\theta_s = 0.38$, $\lambda = 0.50$, $K_{sat} = 1.2 \times 10^{-5}$ m/s, and $dh/dz = 0$.

- (a) (4 points) In this 1D vertical-flow setting, identify the driving force for water flow under the condition above.
- (b) (8 points) Plot $k_{rw}$ and $q_w$ as functions of water saturation, $S_w = \theta/\theta_s$, over $S_w \in [\theta_r/\theta_s, 1]$. Clearly label axes and units.
- (c) (8 points) Based on your plots from (b), explain why direct measurement of relative permeability curves is difficult.

5. (30 points) Richards assumptions and Richards equation.

- (a) (6 points) Starting from the two-phase air-water equations, state the key assumptions used to derive the Richards equation for variably saturated water flow in the vadose zone.
- (b) (6 points) If the pressure in the air phase of an unsaturated soil remains essentially at atmospheric pressure, is it then reasonable to conclude that the air does not move? Why or why not? Does this type of air movement violate the Richards assumptions?
- (c) (6 points) When deriving the Richards equation, if air density cannot be considered negligible compared to water density, how would you modify the Richards equation or functions associated with it to account for that?
- (d) (6 points) Suppose you wish to write a mathematical model for water flow in the unsaturated zone that can also include zones of full saturation. Of the three equations (mixed, $h$-based, and $\theta$-based), which is most appropriate? Which (if any) is not appropriate? Explain your answers.
- (e) (6 points) Give one field scenario where the Richards assumptions may fail (or become weak), and explain the expected modeling consequence.