HWRS 561b: Physical Hydrogeology II

Porous medium models and macroscopic description (Part 2)

Agenda:

1. Soil water characteristics (SWC)
2. Leverett J function; Miller scaling
3. Unsaturated permeability and relative permeability

Review

- Bundle of cylindrical tubes model
- Advanced porous medium models
  - Bundle of triangular tubes model
  - Pore-network model
  - Physical porous medium models: micro-channels
  - Direct imaging of soil and rock pore structures
  - Extract pore-network from digital soil/rock images
- SWC: measurement, physical meaning, mathematical description

Mathematical Description of SWC

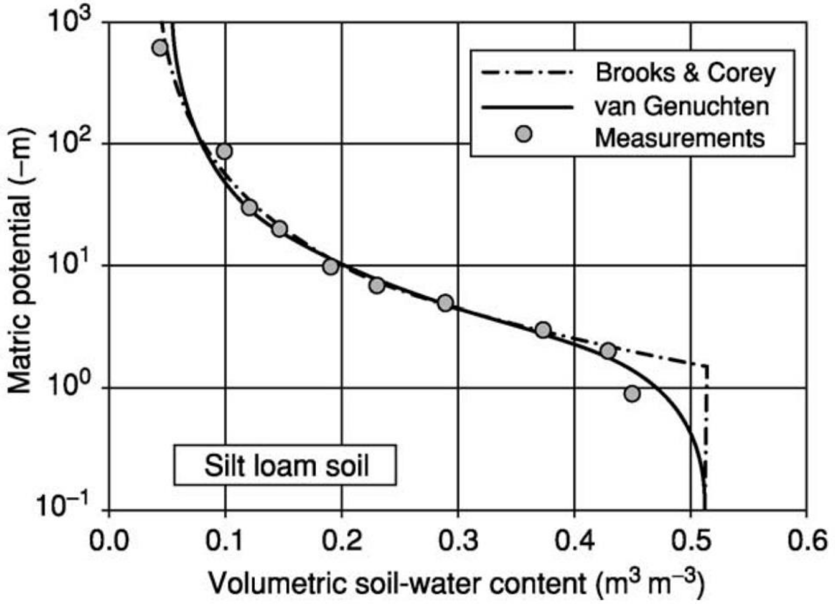
Brooks-Corey (1964)

$p_c = p_d s_e^{-1/\lambda}$

$s_e = (s_w - s_{w,r})/(1 - s_{w,r})$

p_d is entry pressure

λ is a parameter related to pore size distribution



Notes:
(1) "λ" was used in B-C's original paper. Ty's notes used "L" for "λ".
(2) $s_e = (p_c/p_d)^{-\lambda}$
(3) Non-differentiable at $p_c = p_d$

van Genuchten (1980)

$p_c = \frac{1}{\alpha}(s_e^{-1/m} - 1)^{1/n}$

α is a parameter *related* (*NOT* equal) to $1/p_d$.

n is a parameter related to pore size distribution

$m = 1 - 1/n$ based on the Mualem assumption

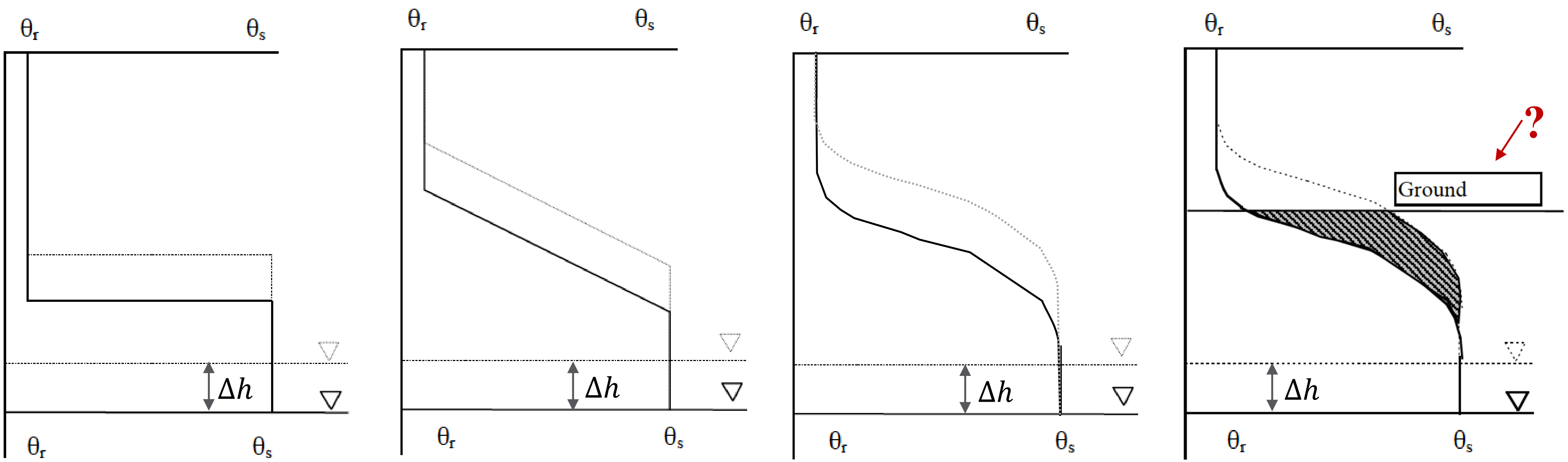
Table 1 Typical van Genuchten model parameters (α, *n*) including residual (θ_r) and saturated (θ_s) water contents compiled from the UNSODA database

Textural class	*N*	θ_r ($cm^3\ cm^{-3}$)	θ_s ($cm^3\ cm^{-3}$)	α (cm^{-1})	n
Sand	126	0.058	0.37	0.035	3.19
Loamy sand	51	0.074	0.39	0.035	2.39
Sandy loam	78	0.067	0.37	0.021	1.61
Loam	61	0.083	0.46	0.025	1.31
Silt	3	0.123	0.48	0.006	1.53
Silt loam	101	0.061	0.43	0.012	1.39
Sandy clay loam	37	0.086	0.40	0.033	1.49
Clay loam	23	0.129	0.47	0.030	1.37
Silty clay loam	20	0.098	0.55	0.027	1.41
Silty clay	12	0.163	0.47	0.023	1.39
Clay	25	0.102	0.51	0.021	1.20

N, the number of soils or samples of a given textural class from which the mean values are compiled.
Reproduced from Leij FJ, Alves WJ, van Genuchten MT, and Williams JR (1996) *The UNSODA Unsaturated Hydraulic Database*. EPA/600/ R-96/095. Cincinnati, OH: US Environmental Protection Agency.

Tuller & Or (2004)

Specific Yield and Drainable Porosity

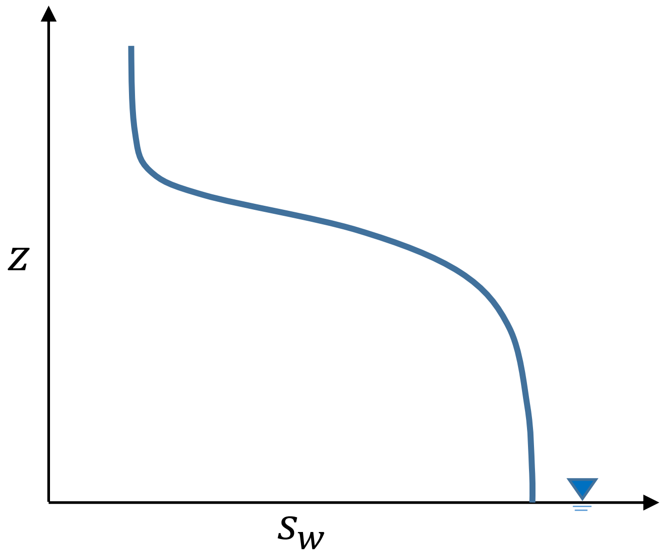


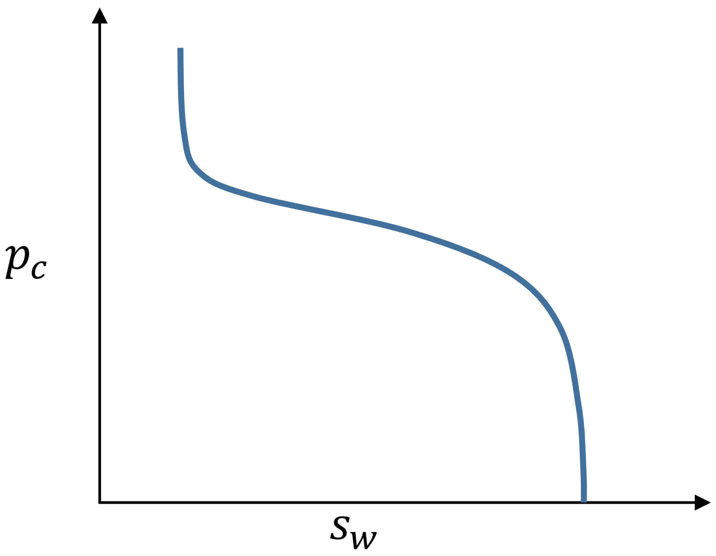
Volume of water release: $(\theta_s - \theta_r)\Delta h$ $(\theta_s - \theta_r)\Delta h$ $(\theta_s - \theta_r)\Delta h$ $< (\theta_s - \theta_r)\Delta h$

- Volume of water release is independent of water content distribution if the capillary transition zone is below the ground surface, if θ_s and θ_r remain unchanged.
- How about the time scale for the water release?

Hydrostatic Water Saturation Distribution

Water saturation distribution under hydrostatic conditions



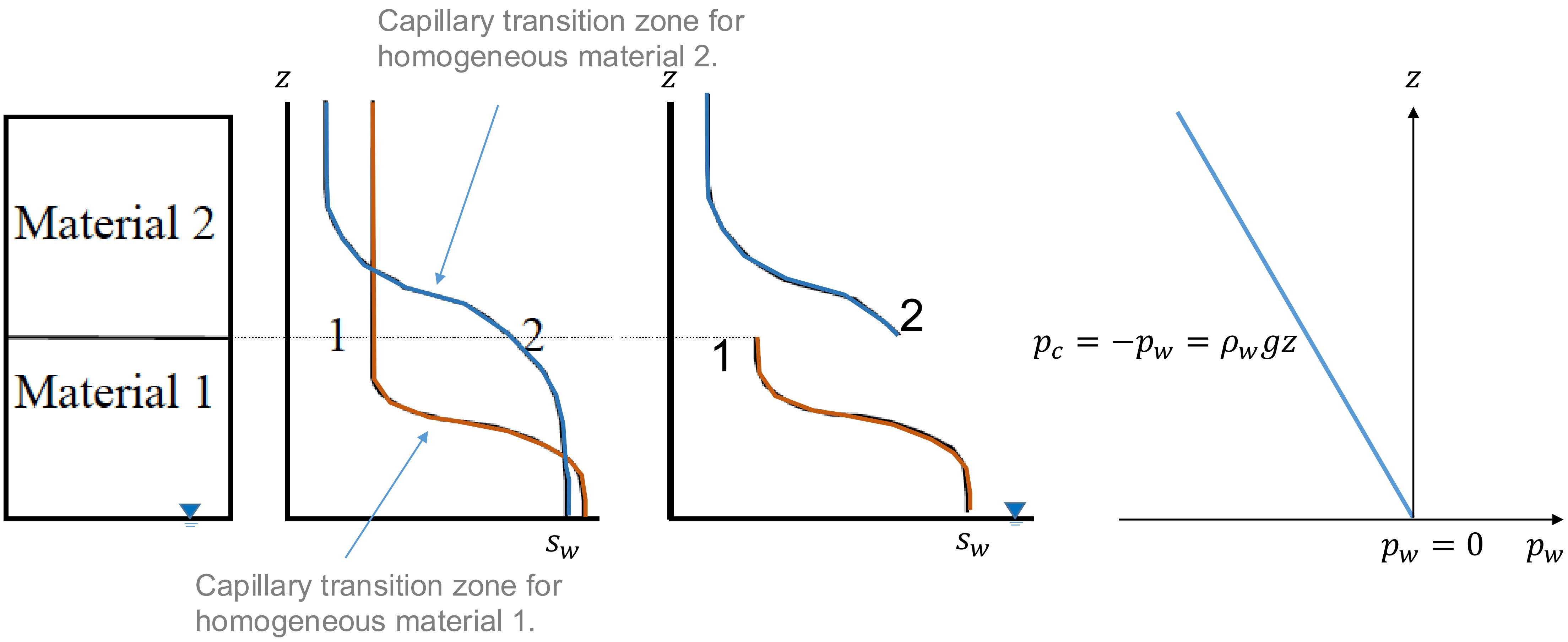


Hydrostatic$\Rightarrow$ $\left.\begin{aligned} p_w(z) &= 0 - \rho_w g z \\ p_w &= 0 - p_c = -p_c(S_w) \end{aligned}\right\} \Rightarrow p_c(S_w) = \rho_w g z \Rightarrow p_c(S_w) \sim z$

- $z(S_w)$ and $p_c(S_w)$ have the same shape. They can be rescaled ("stretched" or "compressed") along the vertical axis to overlap with each other.

Hydrostatic Water Saturation Distribution

Hydrostatic unsaturated layered systems



- S_w is <u>discontinuous</u> at the material interface, but the p_c and p_w or ψ_w <u>are continuous</u>.

Leverett J function/scaling

Capillary Behavior in Porous Solids

BY M. C. LEVERETT,* MEMBER A.I.M.E.

(Tulsa Meeting, October 1940)

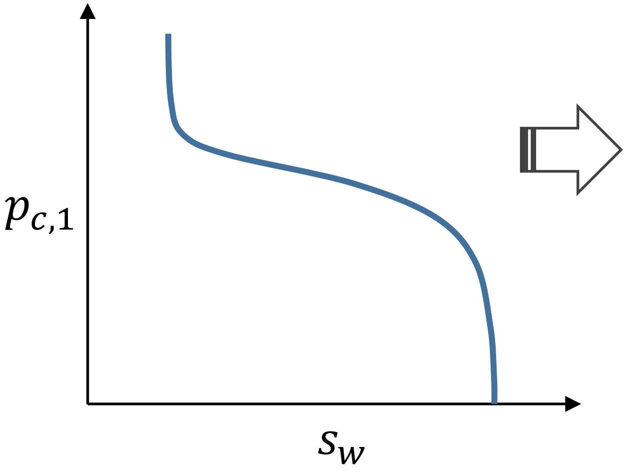
Manuscript received at the office of the Institute June 14, 1940. Issued as T.P. 1223 in PETROLEUM TECHNOLOGY, August 1940.

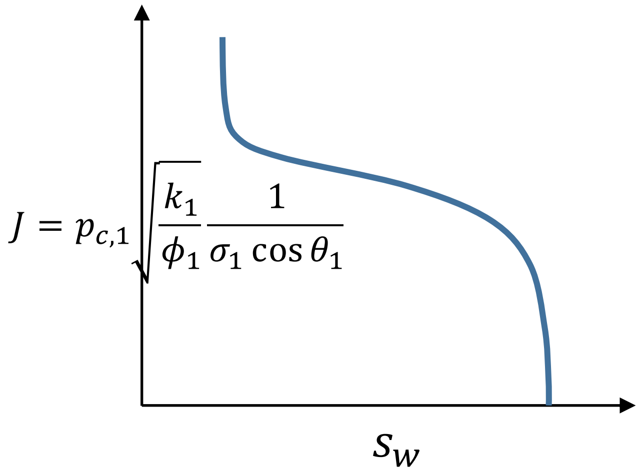
* Humble Oil and Refining Co., Houston, Texas.

$$p_c \sim \frac{\sigma \cos\theta}{r}$$

$$k \sim \phi r^2$$

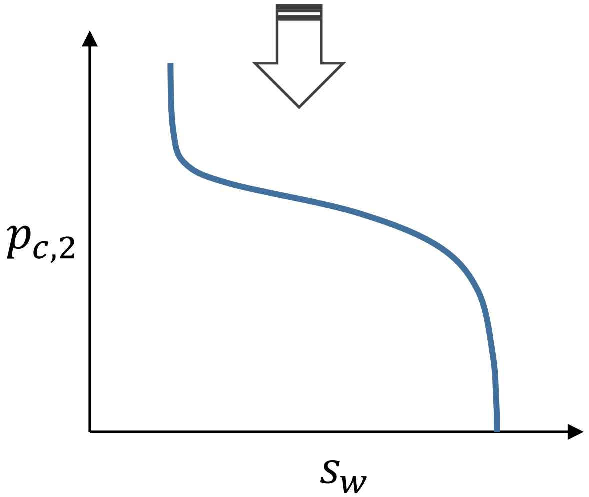
$$\Rightarrow p_c \sim \frac{\sigma \cos\theta}{\sqrt{\frac{k}{\phi}}} \Rightarrow p_c = \sqrt{\frac{\phi}{k}}\, \sigma \cos\theta \, J(s_w)$$





$$J = p_{c,1}\sqrt{\frac{k_1}{\phi_1}}\frac{1}{\sigma_1 \cos\theta_1} = p_{c,2}\sqrt{\frac{k_2}{\phi_2}}\frac{1}{\sigma_2 \cos\theta_2}$$

$$p_{c,2} = \frac{J}{\sqrt{\frac{k_2}{\phi_2}}\frac{1}{\sigma_2 \cos\theta_2}} = \frac{p_{c,1}\sqrt{\frac{k_1}{\phi_1}}\frac{1}{\sigma_1 \cos\theta_1}}{\sqrt{\frac{k_2}{\phi_2}}\frac{1}{\sigma_2 \cos\theta_2}}$$



- Leverett J function allows one to derive the SWC for a porous medium using the SWC from a similar porous medium + some basic hydraulic information (permeability and porosity)

Miller Scaling

JOURNAL OF APPLIED PHYSICS APRIL, 1956

Physical Theory for Capillary Flow Phenomena

E. E. MILLER, *Department of Physics, and Physics-Consultant, Agricultural Experiment Station, University of Wisconsin, Madison, Wisconsin*

AND

R. D. MILLER, *Department of Agronomy, Cornell University, Ithaca, New York*

(Received August 8, 1955; revised version received December 1, 1955)

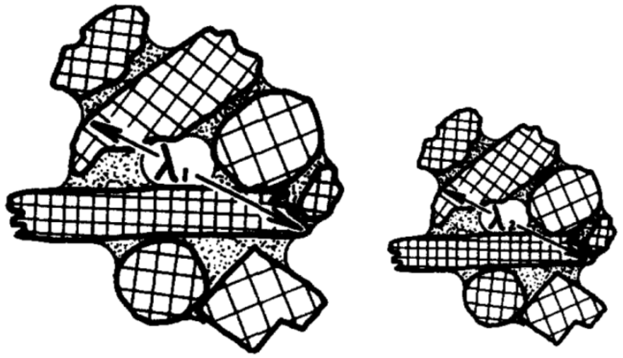


FIG. 2. Illustration of two "similar media" in "similar states." Note that the two characteristic lengths, λ_1 and λ_2, connect corresponding points in the two media.

Dimensionless capillary pressure
(exact same idea as Leverett J scaling)

$$p_{c,0} = \frac{\lambda p_c}{\sigma} \Rightarrow \frac{\lambda_1 p_{c,1}}{\sigma_1} = \frac{\lambda_2 p_{c,2}}{\sigma_2} \Rightarrow p_{c,2} = \frac{\lambda_1 \sigma_2}{\lambda_2 \sigma_1} p_{c,1}$$

$$K_0 = \frac{\mu K}{\rho g \lambda^2} \Rightarrow \frac{\mu_1 K_1}{\rho_1 g \lambda_1^2} = \frac{\mu_2 K_2}{\rho_2 g \lambda_2^2} \Rightarrow K_2 = \frac{\rho_2 \lambda_2^2}{\rho_1 \lambda_1^2} \frac{\mu_1}{\mu_2} K_1$$

Dimensionless conductivity

- Miller scaling allows one to derive SWC and conductivity for a porous medium using SWC and conductivity from a similar porous medium + some basic pore size information

Unsaturated Permeability

Single-phase Darcy's Law

$$\boldsymbol{q}_w = -\frac{\mathbf{k}}{\mu_w}\boldsymbol{\nabla}(p_w + \rho_w g z)$$

$$= -\mathbf{K}_{sat}\boldsymbol{\nabla}H$$

$$H = \frac{p_w}{\rho_w g} + z$$

$$\mathbf{K}_{sat} = \frac{\mathbf{k}\rho_w g}{\mu_w}$$

How about water flow in the presence of air in the porous medium?

Bundle of cylindrical capillary tubes model

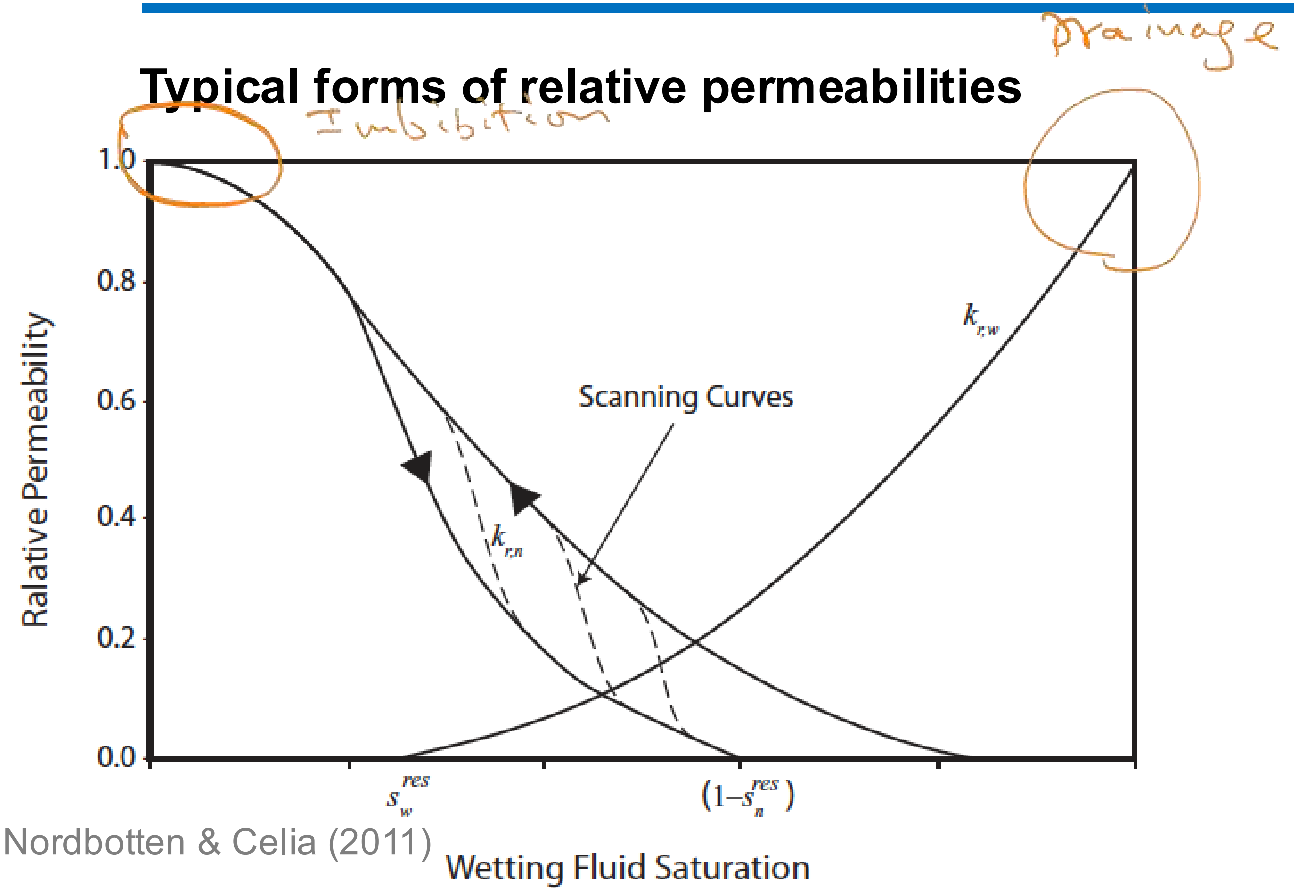
Extended Darcy's Law for unsaturated flow (Buckingham, 1907)

$$\boldsymbol{q}_w = -\frac{k_{r,w}(S_w)\mathbf{k}}{\mu_w}\boldsymbol{\nabla}(p_w + \rho_w g z)$$

$$= -k_{r,w}(S_w)\mathbf{K}_{\mathrm{sat}}\boldsymbol{\nabla}H$$

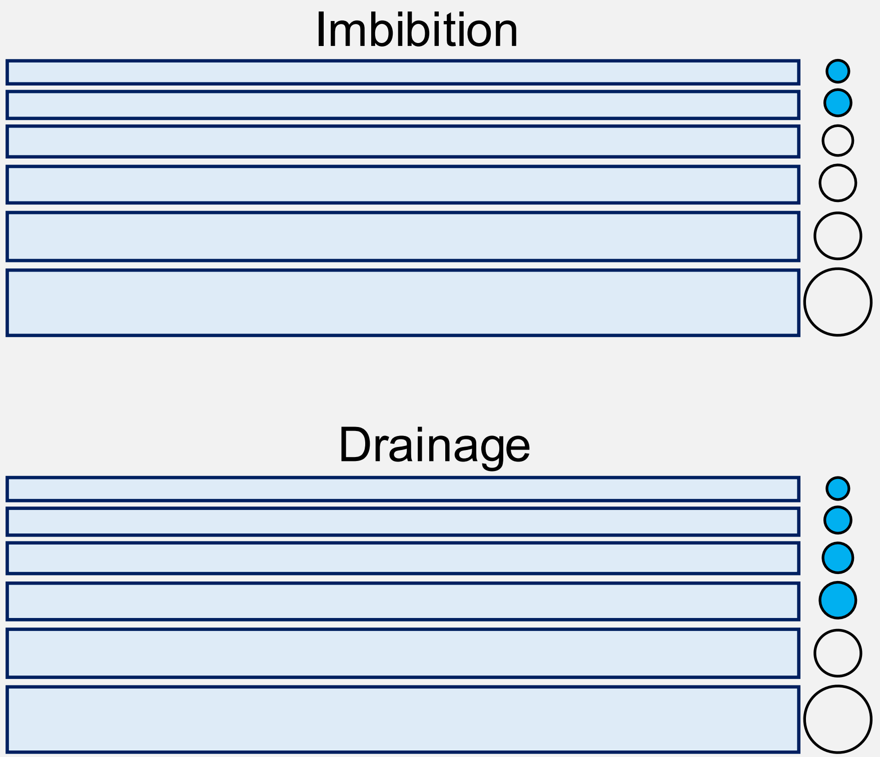
Relative Permeability

Typical forms of relative permeabilities



Nordbotten & Celia (2011)

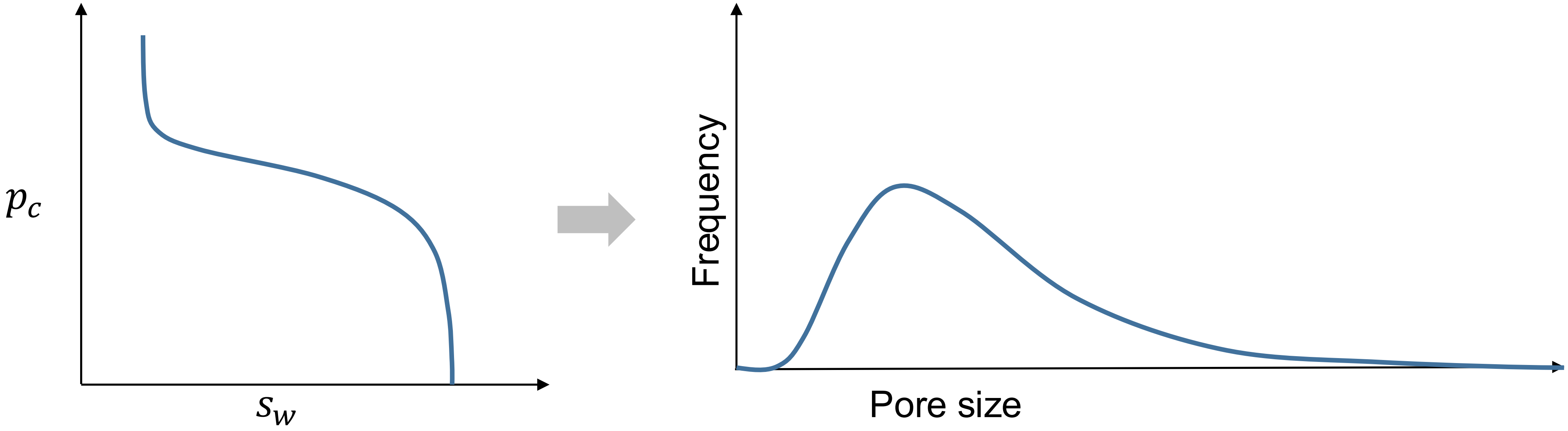
Bundle of cylindrical capillary tubes model



Note: Direct measurement of relative permeability curves is difficult and time-consuming.

Estimate Relative Permeability from SWC

1. Extraction of radii distribution of the Bundle of Cylindrical Capillary tubes (based on BCC)



2. For a given p_c or s_w, compute the fluxes for the cylindrical tubes filled by water ($Q = \frac{\pi r^4}{8\mu_w}\frac{\Delta p}{L}$) and sum them up to obtain the to flux, and then to compute the overall conductivity of the bundle.

Estimate Relative Permeability from SWC

HYDRAULIC PROPERTIES OF POROUS MEDIA

by

R. H. Brooks and A. T. Corey

March 1964

Google scholar citation: ~10,000

Following the Burdine approach, a theory is presented that develops the functional relationships among saturation, pressure difference, and the permeabilities of air and liquid in terms of hydraulic properties of partially saturated porous media. Procedures for determining these hydraulic properties from capillary pressure - desaturation curves are described. Air and liquid permeabilities as a function of saturation and capillary pressure are predicted from the experimentally determined hydraulic properties.

tional relationship among effective permeability, saturation and capillary pressure is essential. This paper develops these functional relationships using a model of porous media developed by investigators [3, 4, 11, 19] in the petroleum industry. Specifically, the analysis presented here is an extension of the work of Burdine [3].

Based on a bundle of tubes with irregular cross sections.

WATER RESOURCES RESEARCH

VOL. 12, NO. 3 JUNE 1976

Google scholar citation: ~10,000

A New Model for Predicting the Hydraulic Conductivity of Unsaturated Porous Media

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A simple analytic model is proposed which predicts the unsaturated hydraulic conductivity curves by using the moisture content–capillary head curve and the measured value of the hydraulic conductivity at saturation. It is similar to the Childs and Collis-George (1950) model but uses a modified assumption concerning the hydraulic conductivity of the pore sequence in order to take into account the effect of the larger pore section. A computational method is derived for the determination of the residual water content and for the extrapolation of the water content–capillary head curve as measured in a limited range. The proposed model is compared with the existing practical models of Averjanov (1950), Wyllie and Gardner (1958), and Millington and Quirk (1961) on the basis of the measured data of 45 soils. It seems that the new model is in better agreement with observations.

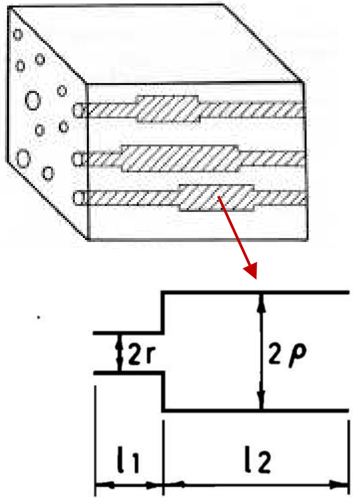

Fig. 1. A combination of cylindrical tubes used to evaluate the hydraulic conductivity of a pair of capillary elements.

A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils[1]

M. TH. VAN GENUCHTEN[2]

Google scholar citation: ~30,000

ABSTRACT

A new and relatively simple equation for the soil-water content-pressure head curve, $\theta(h)$, is described in this paper. The particular form of the equation enables one to derive closed-form analytical expressions for the relative hydraulic conductivity, K_r, when substituted in the predictive conductivity models of N.T. Burdine or Y. Mualem. The resulting expressions for $K_r(h)$ contain three independent parameters which may be obtained by fitting the proposed soil-water retention model to experimental data. Results obtained with the closed-form analytical expressions based on the Mualem theory are compared with observed hydraulic conductivity data for five soils with a wide range of hydraulic properties. The unsaturated hydraulic conductivity is predicted well in four out of five cases. It is found that a reasonable description of the soil-water retention curve at low water contents is important for an accurate prediction of the unsaturated hydraulic conductivity.

Additional Index Words: soil-water diffusivity, soil-water retention curve.

van Genuchten, M. Th. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 44:892–898.

the unsaturated hydraulic conductivity have also been developed. For example, Brooks and Corey (1964) and Jeppson (1974) each used an analytical expression for the conductivity based on the Burdine theory (Burdine, 1953). Brooks and Corey (1964, 1966) obtained fairly accurate predictions with their equations, even though a discontinuity is present in the slope of both the soil-water retention curve and the unsaturated hydraulic conductivity curve at some negative value of the pressure head (this point is often referred to as the bubbling pressure). Such a discontinuity sometimes prevents rapid convergence in numerical saturated-unsaturated flow problems. It also appears that predictions based on the Brooks and Corey equations are somewhat less accurate than those obtained with various forms of the (modified) Millington-Quirk method.

Recently Mualem (1976a) derived a new model for predicting the hydraulic conductivity from knowledge of the soil-water retention curve and the conductivity at saturation. Mualem's derivation leads to a simple integral formula for the unsaturated hydraulic conductivity which enables one to derive closed-form

Relative Permeability

Brooks-Corey (1964)

$$k_{r,w} = s_e^{(2+3\lambda)/\lambda}$$

$$k_{r,nw} = (1 - s_e)^2 (1 - s_e^{(2+\lambda)/\lambda})$$

λ is a parameter related to pore size distribution

van Genuchten (1980) – Mualem (1976)

$$k_{r,w} = s_e^{1/2} \left[1 - \left(1 - s_e^{1/m} \right)^m \right]^2$$

$k_{r,nw}$ was not proposed by V-G. Often the $k_{r,nw}$ from B-C is used

α is a parameter related to the inverse of entry pressure

n is a parameter related to pore size distribution

$m = 1 - 1/n$ based on the Mualem assumption